



Artificial reef placement: a red snapper, *Lutjanus campechanus*, ecosystem and fuzzy rule-based model

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Abstract This study on reef placement involves two aspects: (1) the development of a bioenergetics model for red snapper, *Lutjanus campechanus* (Poey), in Gulf of Mexico waters off the coast of Alabama, where significant numbers of artificial reefs are thought to exist, and (2) a fuzzy rough set model by which parameters determined from the bioenergetics model can provide a decision tool for optimally spacing artificial reefs during deployment. The bioenergetics and consumption rates of *L. campechanus* by age class foraging on artificial reefs in the Gulf of Mexico off the coast of Alabama provided input into the fuzzy rule-based model. After conducting multiple simulations, highest certainty in optimal reef spacing was achieved for reef distances between 0.50 and 0.95 km such that no more than two fit within a 1-km² area. Results can inform fisheries managers about placement of artificial reefs to affect the health and survival of reef-associated species.

KEYWORDS: bioenergetics, fisheries management, fuzzy set theory, spatial arrangement.

Introduction

Bioenergetics models require two basic data sets: (1) estimates of *P*-value and consumption (g) in the bioenergetics budget equation as functions of temperature and body size; and (2) site-specific data, such as ambient water temperature, growth estimates of the species and diet (Kitchell *et al.* 1977). Thus, bioenergetics provides a theoretical framework for relating growth rates and feeding rates of an organism to environmental conditions, and provides some insight into causal relationships among these variables (Allen & Wootton 1982). Population consumption is estimated by knowing the diet, respiration rate and growth rate (Kitchell *et al.* 1977; Ney 1990; Hanson *et al.* 1997; Bajer *et al.* 2004a) of an average individual coupled with estimates of population size (Hayward & Margraf 1987; Hill & Magnuson 1990). A bioenergetics model can determine whether prey production (Jones *et al.* 1993; Rand *et al.* 1995; Kershner *et al.*

1999; Yamamura 2004) is sufficient to account for observed growth rates, or it can estimate how much food is consumed to result in those growth rates, hence a greater understanding of the ecology of the species in the habitat in which it lives (Kitchell *et al.* 1977).

Many bioenergetics studies have used the Wisconsin bioenergetics model (also known as Fish Bioenergetics 3.0; and hereafter as Bioen95), which has been used for a wide variety of fishes (Kitchell *et al.* 1977; Ney 1993). This model uses an energetics-based approach focused upon the processes that regulate growth in individual fishes (Kitchell *et al.* 1977). Bioen95 assembles individuals into age and size population classifications, separates natural and fishing mortality rates, and specifies the trophic ontogeny of predator–prey interactions. The software is accessible, as it employs data most frequently collected by biologists, including habitat, thermal history, size at age, growth curves, stomach contents, size at sexual maturity and mortality rates. Yet, in an ecological investigation, perfect

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knowledge is rarely, if ever, available because natural systems do not conform to crisp definitions (Mackinson *et al.* 1999).

The use of crisp (not fuzzy) sets requires the expert to establish sustainability thresholds for attributes, measure the attributes and determine whether measured attributes attain or fall short of the thresholds. This approach assumes that the expert can make a sharp, unambiguous distinction that is incompatible with the numerous uncertainties in ecosystem assessments. A fuzzy logic approach overcomes the conventional approach according to Prato (2005). Compared with conventional knowledge-based systems, fuzzy set theory offers better representation and processing of imprecise data, and of vague knowledge in the form of linguistic rules (Salski 1992). Such a fuzzy rule-based model is characterised by data richness and complexity, thrives in a data-poor environment, is adaptive such that better approximations are possible with more data, requires relatively few rules to describe the data, and can provide patches across traditional curve-based distributions (Mackinson *et al.* 1999). Fuzzy logic has a rigorous mathematical foundation shown not to contradict but to encompass probability theory (Kosko 1990), while dealing with uncertainty where ambiguous terms are present. (Articles in Zadeh 1979, 1981 and 1983 illustrate the use of fuzzy sets.)

Given the high uncertainty and imprecise environment in fisheries science and management, it has been suggested that fishery statistical parameters do not follow deterministic rules but fluctuate around a fuzzy value (May *et al.* 1978). More recently, Koutroumanidis *et al.* (2006) applied fuzzy expected intervals to fishery landings data. As an extension of this work, Sylaios *et al.* (2010) considered the different degrees of membership within a cluster providing a series of five rules related to landings and fish production.

In line with this direction towards fuzzy rule-based models, a rough set theory model (Pawlak 1982; Mrozek 1985; Arciszewski & Ziarko 1986; Fibak *et al.* 1986; Mrozek 1987; Grzymala-Busse 1988; deKorvin & Shipley 1993) using fuzzy sets was developed to investigate artificial reef placement as a function of fish consumption. In the natural environment, many reef-associated fishes forage away from reefs (Bohnsack 1989). As such, distances between shelter and foraging locations can be important determinants of foraging success.

Rules were developed according to: (1) consumption estimates generated by a bioenergetics model for *Lutjanus campechanus* (Poey) describing food requirements based upon empirically determined growth rates; and (2) artificial reef data predicted from

simulating the effect of foraging specifically related to food consumption of *L. campechanus* on the reefs. The fuzzy set model considers distances from maximum to minimum spacing and determines a strength of belief in the certainty or possibility of the hypothesised rules for reef placement. Based upon the strength of belief in a rule, the artificial reef placement location can be accepted or rejected as being conducive to consumption at the reef and foraging behaviour of the fish, specifically *L. campechanus*.

Methods

Bioenergetics model for Lutjanus campechanus

Although numerous species have bioenergetics models, a model for *L. campechanus* has not previously been developed. To model a new species, a set of parameters must be developed. These can be derived from published reports, estimated from specifically designed field or laboratory studies, or borrowed from closely related species. Although species borrowing data has met with some criticism (Ney 1990, 1993), the positive viewpoint is that using parameters from closely related species or those with similar morphologies and life-history attributes provides a reasonable modelling assumption until actual ecological species-specific parameters can be derived. As such, parameter ranges for other warm water omnivores that consume some indigestible prey available from the Bioen95 manual (decapod crustaceans in this instance) were considered suitable for use in the *L. campechanus* model in the absence of *L. campechanus*-specific information. Temperatures in the model were based upon observations made during numerous sampling efforts for *L. campechanus* conducted between 1996 and 2002 on Alabama artificial reefs (Watterson *et al.* 1998; Patterson 1999; Patterson *et al.* 2001a, b; Patterson & Cowan 2003; Jackson *et al.* 2007). The allometric mass function intercept was also adjusted until juvenile *L. campechanus* consumption was between 2% and 4% and adult consumption was between 1% and 3%. Furthermore, as all *L. campechanus* in the model were larger than 10 g the same parameters were used for every age class.

For this study, an assumption was made that approximately 1500 *L. campechanus* would represent a population on a single artificial reef (reef volumes 1.8–3.5 m³). The bioenergetics model was then adjusted to formulate equation-specific input variables for *L. campechanus*. These variables were extracted from empirical data in Patterson *et al.* (2001a) and Jackson *et al.* (2007), which included total length (TL),

age classifications, instantaneous growth in weight, TL to weight comparisons, weight lost due to spawning and total mortality rate.

Although natural mortality rates were assumed constant for all age classes (0.10), fishing mortality affects only those age classes where the fish is large enough to be legal for capture (> 406 mm TL). Fishing mortality (F) was calculated using two different year classes from data in Patterson *et al.* (2001a). The catch-at-age between ages 5 and 11 was used to estimate F at 0.43 per year, which is close to the F -estimate (0.46 per year) for the same age classes derived from the most recent stock assessment for *L. campechanus* in the eastern Gulf (SEDAR 7, 2005). Therefore, total mortality rate for fish over the legal limit of 406 mm (≥ 3 age class) was assumed to be 0.53 per year.

To eliminate bias, *L. campechanus* numbers were adjusted to include only non-tournament catches. Next, ages were assigned based upon data from Patterson *et al.* (2001a), and the von Bertalanffy growth function (Patterson *et al.* 2001a) was fitted to the revised catch numbers based upon TL (mm) and age. Age class 1 was excluded from the calculations of biomass because these fish are not fully recruited to the offshore artificial reefs until age 2. Finally, age-class frequencies were determined; all information on size and age is summarised in Table 1.

Total length and weight (WT) were calculated for each age class based on non-tournament catch data using equations from Patterson *et al.* (2001a):

$$TL = 1181 \times (1 - e^{-0.120(t+0.652)}) \quad (1)$$

$$WT = (4.68 \times 10^{-9}) TL^{3.17}, \quad (2)$$

where $2 \leq t \leq 34$. The calculated weight for an individual fish in each age class was multiplied by the number of fish in the cohort to equal the mass of the starting population (Table 2), assuming no sexual dimorphism in size and growth rate (Patterson *et al.* 2001a; Fischer 2007).

Lutjanus campechanus reaches maturity at 208–309 mm TL (Patterson *et al.* 2001a; Woods 2003). Thus, half of the fish in age class 2 and all the fish in the subsequent age classes were considered sexually mature and subject to gamete shedding during spawning (Woods 2003). While *L. campechanus* spawns from June to August (Render 1995), the Bioen95 software only allows for 1 day of spawning as it relies on a stepwise progression of time to represent maturity to death of the species within the ecosystem. The model compresses the environment such that all gametes that are lost during the summer months must therefore be represented during the single day selected. Day 201 (July 20) was chosen to represent this single day of spawning based upon the reported mid-July peak in spawning by *L. campechanus* (Woods 2003). As females lose more weight during gamete shedding than males, only the female weight shedding was taken into account. Weight lost due to spawning was calculated by multiplying the total estimated annual fecundity for

Table 1. Size class and age group frequencies for *Lutjanus campechanus* bioenergetics input

Size (mm)	Number of fish sampled	Number of tournament fish excluded	TL (mm) frequencies	Age	Age-class frequency
200–250	20		0.0114	1	0.0114
250–300	65		0.0370	2	0.1367
300–350	175		0.0997		
350–400	145		0.0823	3	0.2023
400–450	210		0.1197		
450–500	250	10	0.1637	4	0.2967
500–550	200	5	0.1330	5	0.2319
550–600	160	15	0.0989		
600–650	120	20	0.0682	6	0.0921
650–700	60	25	0.0239		
700–750	45	25	0.0136	7	0.0136
750–800	65	50	0.0102	8	0.0102
800–850	55	45	0.0068	9	0.0068
850–900	40	35	0.0034	≥ 10	0.0143
900–950	25	20	0.0034		
950–1000	10	9	0.0007		
$N = 1466$					

each age class by the estimated weight of an individual egg (25 µg; Houde 1989) and by 0.50 to represent only the females in the age classes. Production of reproductive tissue occurs during normal growth and loss occurs during spawning. If a bioenergetic model run includes spawning data for mature fish, a user-defined proportion of fish mass will be lost on the selected spawning day. While separate runs can be done for the males and females, it is generally sufficient to estimate the average proportion of gonad production lost for both sexes combined. Variation in water temperature in the Gulf of Mexico, where the artificial reefs systems are located, was determined from past research (Strelcheck 2001; McCawley 2003; McCawley & Cowan 2007) to represent changes in temperature across seasons. Seasonal water temperature, as well as a minimum temperature below which *L. campechanus* cannot survive, was used in the model. For *L. campechanus* on the Alabama artificial reefs, the minimum temperature was set at 14 °C.

Finally, diet data were analysed based on McCawley (2003) and McCawley and Cowan (2007), which identified seven main categories of prey species in Alabama coastal waters. Fish species (19.5%), crab

species (20.3%), pelagic zooplankton (8.0%) and adult mantis shrimp (12.6%) were the principal components of the *L. campechanus* diet (McCawley 2003; McCawley & Cowan 2007) and included in this study.

Using the taxonomic breakdown of stomach contents from 300 to 499 mm FL *L. campechanus* collected on Alabama artificial reefs by season [presented in table 3 by McCawley and Cowan (2007)], the dietary composition for juvenile and adult *L. campechanus* was determined by season (Table 3).

For this study, calorific density of the prey items upon which *L. campechanus* feed was calculated from appendix 2 in McCawley and Cowan (2007). While McCawley and Cowan (2007) presented both the values that they determined using bomb calorimetry as well as data from the literature, this model used average calorific density for each prey group based only upon the calorimetric data (Table 4).

Bomb calorimetry studies were also used to determine how much of each prey item could be digested by *L. campechanus* (McCawley & Cowan 2007). Hard-shelled prey items, such as crab, cannot be digested easily, unlike other prey items with small bones or surface area. As all of the mass of many prey items cannot be digested by *L. campechanus*, specific proportions of the prey were assumed to be indigestible and the percentage was estimated from presence of bone structures or hard shell (Table 5).

Each individual cohort file representing different age classes was run in Bioen95 for a total of 365 days. Predicted values (i.e. realised daily consumption as a fraction of C_{max}) were calculated from the pre-defined start and final weights (Table 6). Additional details about the *L. campechanus*, bioenergetics model can be found in Shipley (2008).

Lutjanus campechanus ecosystem fuzzy rule-based model

The main idea behind the concept of a fuzzy set is Zadeh's (1965) generalisation of the concept of the characteristic function of a set which he renamed membership function. When a fuzzy subset *A* of a set *X*

Table 2. Total length and weight calculated for each *Lutjanus campechanus* (Poey) age class

Age	Total length (mm)	Weight (kg)
1	212.38	0.1115
2	321.91	0.4166
3	419.05	0.9612
4	505.21	1.7388
5	581.63	2.7175
6	649.41	3.8542
7	709.52	5.1027
8	762.83	6.4201
9	810.12	7.7687
≥10	932.17	12.1212

Values for ≥10 age class were calculated from averaging the calculated total lengths from age classes 10–15 and using that to calculate weight.

Table 3. Diet proportions of major prey categories for juvenile and adult snapper accounting for seasonal variation

Day of simulation	Fish species		Crab species		Pelagic zooplankton		Mantis shrimp	
	Juvenile	Adult	Juvenile	Adult	Juvenile	Adult	Juvenile	Adult
1	0.23	0.20	0.20	0.30	0.25	0.15	0.32	0.35
60	0.32	0.28	0.20	0.25	0.45	0.21	0.03	0.26
152	0.27	0.25	0.38	0.40	0.15	0.09	0.20	0.26
244	0.28	0.29	0.29	0.35	0.28	0.06	0.15	0.30
365	0.23	0.20	0.20	0.30	0.25	0.15	0.32	0.35

is a function of X into $[0,1]$ and set X is finite, say $X = \{x_1, x_2, \dots, x_n\}$, then a fuzzy set A can be written as $A = \sum_{i=1}^n a_i/x_i$, where a_i represents the belief that the value of A is x_i and x_i may be expressed in qualitative terms. A is sometimes called the membership function such that a_i is the belief that x_i belongs to A . If A is a standard (not fuzzy) subset of X , say $A = \{x_i1, x_i2, \dots, x_ik\}$, then A can be represented as $A = \sum_{p=1}^k a_{ip}/x_{ip}$, where $a_{ip} = 1$. This states that the belief that x_{ip} belongs to A equals one. A fuzzy set, therefore, is a generalisation of a standard set since a_i does not have to be equal to one; $0 \leq a_i \leq 1$ (Jain 1977; Dubois & Prade 1979).

Table 4. Mean caloric density values for the principal prey items of *Lutjanus campechanus* (Poey) collected on Alabama artificial reefs (McCawley & Cowan 2007)

	Caloric density (cal g ⁻¹) dry wt
Fish species	4947.49
Crab species	3138.45
Pelagic zooplankton	3511.23
Adult mantis shrimp	3894.01

Table 5. Percent of each prey item that is considered indigestible for *Lutjanus campechanus* (Poey)

Prey items	Percent indigestible
Fish	20
Crab	40
Pelagic zooplankton	12
Mantis shrimp	15

Table 6. Initial and final parameter estimates from a Bioen95 model for *Lutjanus campechanus* (Poey) ages classes on a single artificial reef after a single year model run

Age	Start weight* (g)	Final weight* (g)	Run P-value	Initial population*	Final population	% spawned on day 201*	Consumption (g)
2	416.6	961.2	0.8691	205	184	1.06	4253.9
3	961.2	1738.8	0.7316	304	155	5.54	5934.4
4	1738.8	2717.5	0.7306	445	227	6.78	8581.3
5	2717.5	3854.2	0.7299	348	178	7.75	11 403.1
6	3854.2	5102.7	0.7499	138	70	11.93	14 677.3
7	5102.7	6420.1	0.7265	20	10	8.76	17 111.5
8	6420.1	7768.7	0.7236	15	7	8.91	19 836.7
9	7768.7	12 121.2	0.9292	10	5	8.73	32 296.4
10+	12 121.2	15 161.1	0.7856	22	11	5.59	34 628.6

*Input values for each age class.

If A and B are two fuzzy sets, the operations 'union', 'intersection' and 'complement' can be defined as:

$$\text{If } A = \sum_{i=1}^n \frac{a_i}{x_i} \text{ and } B = \sum_{i=1}^n \frac{b_i}{x_i},$$

$$\text{then } A \vee B = \sum_{i=1}^n \frac{(a_i \vee b_i)}{x_i} = \sum_{i=1}^n \frac{\text{Max}\{a_i, b_i\}}{x_i} \quad (3)$$

$$A \wedge B = \sum_{i=1}^n \frac{(a_i \wedge b_i)}{x_i} = \sum_{i=1}^n \frac{\text{Min}\{a_i, b_i\}}{x_i} \quad (4)$$

$$\text{and } A' = \sum_{i=1}^n \frac{(1 - a_i)}{x_i}. \quad (5)$$

For a general discussion of the fuzzy logic concepts above, see Kaufmann and Gupta (1985), Klir and Folger (1988) and Zadeh (1965, 1975).

Extension principles (see Dubois & Prade 1980 and Zebda 1984) for fuzzy sets consider f as a function from X into Y , with Y as any set and A as above, such that f can be extended to fuzzy subsets of X by:

$$f(A) = \sum_y \frac{u_{f(A)}(y)}{y}, \quad (6)$$

$$\text{where } u_{f(A)}(y) = \text{Max } A(x), \quad x \in f^{-1}(y)$$

Thus, $f(A)$ is a fuzzy subset of Y . In particular, if f is a mapping from a Cartesian product such as $X \times Y$ to any set, Z , then f can be extended to objects of the form (A, B) where A and B are fuzzy subsets of X and Y by:

$$f(A, B) = \sum_z u_{f(A, B)}(z)/z, \quad (7)$$

where $u_{f(A, B)}(z) = \text{Max Min } \{A(x), B(y)\}, \quad (x, y) \in f^{-1}(z)$.

Rough sets allow inference of knowledge by extraction of certain and possible rules and a measurement of how the values of attributes determine an action (Pawlak 1982, 1983, 1985; Grzymala-Busse 1988). Fuzzy rough set notation transitions the basic rough set theory as defined below (deKorvin *et al.* 1992, 1994; Shipley & deKorvin 1995).

A fuzzy subset A of U is defined by a characteristic function $\mu_A: U \rightarrow [0,1]$. The notation $\Sigma \alpha_i/x_i$ ($0 \leq \alpha_i \leq 1$) denotes a fuzzy subset whose characteristic function at x_i is α_i . If A and B are fuzzy subsets, union, intersection and complement are defined by $\text{Min}\{\mu_A(x), \mu_B(x)\}$, $\text{Max}\{\mu_A(x), \mu_B(x)\}$ and $1 - \mu_A(x)$, respectively. The implication $A \rightarrow B$ is defined by the corresponding characteristic function $\text{Max}\{1 - \mu_A(x), \mu_B(x)\}$ (see, for example, Zadeh 1965, 1968, 1973).

Two functions are defined as pairs of fuzzy sets that will be the input into the rule-based decision.

$$I(A \subset B) = \inf_x \text{Max}\{1 - \mu_A(x), \mu_B(x)\} \quad (8)$$

$$J(A \# B) = \text{Max}_x \text{Min}\{\mu_A(x), \mu_B(x)\}, \quad (9)$$

where A and B denote fuzzy subsets of the same universe. The function $I(A_B)$ measures the degree to which A is included in B and $J(A \# B)$ measures the degree to which A intersects B . If A and B are crisp (non-fuzzy) sets it is easy to establish that $I(A_B) = 1$ if and only if $A \subset B$; otherwise it is zero. Also, in the case of crisp sets $J(A \# B) = 1$ if and only if $A \cap B \neq \emptyset$; otherwise it is zero. The operators I and J yield two sets of rules: the certain rules and the possible rules. The highest level of belief in the certain rules and the highest plausible belief of the possible rules is based upon selection of the threshold of acceptance, α .

The rule structure considered for artificial reef placement was of the form: 'If percentage of maximum consumption at the reef is {High, Low} and foraging consumption is {Great, Small}, then artificial reef distance should be {Major, Minor}', where *Major* and *Minor* reef distances are set within a 1-km² area, because it has been recommended that spacing artificial reefs 600–1000 m from natural reefs is best to minimise fish interaction (Grove & Suno 1985). In addition, Strelcheck *et al.* (2005) found that *L. campechanus* growth rate and site fidelity were inversely related to distance between artificial reefs off Alabama. Thus, the first step in setting up the model was defining the linguistic terms in the rule. First, the antecedent of the rule required defining of High and Low maximum Consumption (P -value, i.e. the realised percentage of physiological maximum consumption), and Great and

Small Consumption (g) from foraging behaviour. The consequent of the rule relied on Maximum and Minimum reef distances.

Growth rates observed at Alabama reefs were used in the bioenergetics model to estimate P -values and the realised weight of food consumed, i.e. consumption (g), for *L. campechanus*. From the data in Table 6, the average P -value across the age classes determined from bioenergetics modelling was 0.78 ± 0.07 , and mean total consumption was $16\,524.5 \pm 10\,514.5 \text{ g yr}^{-1}$ (Table 6). Four scenarios were conducted with the bioenergetics model to determine reasonable percent maximum consumption and foraging consumption estimates indicative of an acceptable range of values per age group (Table 7). Start and end weights for each run varied depending upon combinations of four measures: the lowest recorded size within an age group, the average weight of the age group, the largest recorded weight of the age group and the average weight of the subsequent age group. First, the lowest recorded size at age (cm) was converted to weight (kg) (Patterson *et al.* 2001a) for an age class and used as the start weight, with the average weight of that age for the end weight. The second run used lowest recorded weight as the start weight, but used the largest recorded weight for that age class (Patterson *et al.* 2001a) for the end weight. A third run used the average weight of that age class as the start weight and the largest recorded weight for the end weight. Finally, a fourth run used the average weight for the age class as the start weight and the average weight for the subsequent age class as the end weight. From these runs, the maximum and minimum P -values and consumption values were determined for each *L. campechanus* age group from age 2 through age 10+. Any P -value, and its corresponding consumption value, > 1.1 was discarded as an outlier and thus not statistically usable (Table 7).

The maximum and minimum P -values and consumption (g) per age group from the four runs were used to define triangular distributions typical of fuzzy set-based logic. Crystal Ball (Crystal Ball is a product of Decisioneering Software; <http://www.decisioneering.com>) was used to simulate the triangularly defined distributions through 10 000 runs with input values of maximum, minimum and most probably as generated from the four scenarios. For each age category, the Maximum function was defined to be triangular with minimum P -value, but the likeliest was the maximum P -value which anchored the distribution around the maximum P -values observed. The Minimum function was generated from the minimum P -value and the maximum P -value where the likeliest was selected to be the minimum P -value. In a similar manner, consump-

Table 7. Four scenario runs to determine minimum and maximum *P*-values and consumption (g)

Age	Start weight (g)*	Final weight (g)*	Run <i>P</i> -value [†]	Consumption (g) [†]
2	133.86	416.60	0.8329	2045.1
	133.86	829.42	0.8691	4253.9
	416.60	829.42	0.8130	3736.4
Avg (2:3)	416.60	916.20	0.8691	4253.9
3	333.21	916.20	0.7781	3503.1
	333.21	2999.06		
	916.20	2999.06	1.0206	10 340.2
Avg (3:4)	916.20	1738.80	0.7316	5934.4
4	675.97	1738.80	0.8513	6210.9
	675.97	4888.85		
	1738.80	4888.85	1.0656	16 034.9
Avg (4:5)	1738.80	2717.50	0.7306	8581.3
5	829.42	2717.50	1.0121	9534.7
	829.42	7465.20		
	2717.50	7465.20		
Avg (5:6)	2717.50	3854.20	0.7299	11 403.1
6	1430.90	3854.20	1.0285	13 153.2
	1430.90	9046.98		
	3854.20	9046.98		
Avg (6:7)	3854.20	5102.70	0.7499	14 677.3
7	1682.60	5102.70		
	1682.60	9046.98		
	5102.70	9046.98	0.9605	25 910.2
Avg (7:8)	5102.70	6420.10	0.7265	17 111.5
8	4888.85	6420.10	0.7504	17 415.4
	4888.85	9046.98	0.9862	26 245.6
	6420.10	9046.98	0.8280	24 072.5
Avg (8:9)	6420.10	7768.70	0.7236	19 836.7
9	6420.10*	7768.70	0.7724	19 804.8
	6420.10	10 844.13	0.9613	30 037.9
	7768.70	10 844.13	0.8434	28 057.1
Avg (9:10+)	7768.70	12 121.20	0.9292	32 296.4
10+	9046.98	12 121.20	0.8074	29 554.4
	9046.98	12 871.52	0.8537	31 989.9
	12 121.20	12 871.52	0.6592	27 323.2
Avg (10+:15)	12 121.20	15 161.10	0.7856	34 628.6

*Weight calculation from Patterson *et al.* (2001a) length at age figure yielded a minimum value for Age 9 *Lutjanus campechanus* (Poey) equivalent to maximum value for Age 8 *L. campechanus*. Therefore, the average value for Age 8 *L. campechanus* was used as the minimum value for Age 9 *L. campechanus*. [†]*P*-values > 1.1 were determined to be statistical outliers and with corresponding Consumption (g) were stricken from the data set.

tion for each age class was defined as a triangular function from minimum to maximum with the likeliest value selected based on which type of triangular function was being defined. The values simulated for the Maximum and Minimum fuzzy set type triangular distributions provided expected *P*-values and consumption for each age group that defined the linguistic variables (Table 8).

Actual reef distances were not assumed to be constant for any age group but instead, values were randomly generated [0.01,1] km for each of the age categories through 10 000 simulation runs with Crystal Ball. Reef distances to represent Major (*M*) and Minor (*N*) were also generated through 10 000 runs using Crystal Ball simulation software setting up triangular functions with a minimum of 0.01 km, maximum of 1.00 km, and the likeliest set at 0.50 km (Table 8).

Artificial reef placement distances were allowed to vary to test the optimal location; i.e. those represented by highest certainty and/or possibility of the rules. Nine scenarios were tested for reef distances from 0.01–0.50 to 0.50–0.95 km under the assumptions for Maximum and Minimum consumption at the reef (*P*-values) simulated as described previously using the bioenergetics modelling results for each age group.

Belief in *L. campechanus* realised consumption at the reef (*P*-value) as High or Low, and food consumed (g) as Great or Small was determined. Certainty of the rules was calculated, again based on the 10 000 simulation runs for each designated reef distance. As certainty of any rule approached 100%, the minimum of the range was set (0.50 km) and the maximum was allowed to increase incrementally to 0.95 km.

Instead of using the subjective belief derived from expert opinions, belief was set as the degree of membership of the *P*-values and consumption generated from the bioenergetics modelling to the linguistic variables in the rule antecedent calculated as the ratio of the value to the simulated range of [Maximum, Minimum] for each age class. From Table 8, for example, Age 2 *L. campechanus* *P*-value of 0.8691 surpasses the simulated maximum of the range (i.e. 0.8691/0.8504 = 1.02), so belief in 1.00 that Age 2 *L. campechanus* has High maximum percent consumption at the reef (membership cannot be > 100%). The ratio of the simulated minimum of 0.8317 to the Age 2 *P*-value (0.8317/0.8504) determines its membership in Low as 0.96. Therefore, the membership function for Age 2 is:

$$P - \text{value}_{\text{Age}2} = 1.00/\text{High} + 0.96/\text{Low}$$

and based on 4253.9 g of consumption at Age 2, for the defined range of [3517.6, 2781.4] of 4253.9/3517.6 and 2781.4/4253.9, respectively, determines:

$$\text{Consumption}_{\text{Age}2} = 1.00/\text{Great} + 0.65/\text{Small}$$

again setting the simulated value for Great membership to 1.00 membership.

Table 8. *P*-value, consumption and reef distance values with their corresponding fuzzy set-based simulated minimums and maximums*

Age	<i>P</i> -Value (% of max consumption)		High Low Consumption				Great Small		Reef distance (km)		Major Minor				
	Max.	Min.	(<i>H</i>)	(<i>L</i>)	(g)	Max.	Min.	(<i>G</i>)	(<i>S</i>)	Max.	Min.	(<i>M</i>)	(<i>N</i>)		
2	0.8691	0.8504	0.8317	1.02	0.96	4253.9	3517.6	2781.4	1.21	0.65	0.95	0.50333	0.503	1.89	0.53
3	0.7316	0.9243	0.8279	0.79	1.13	5934.4	8061.2	5782.1	0.74	0.97	0.34	0.50333	0.503	0.68	1.48
4	0.7306	0.9539	0.8423	0.77	1.15	8581.3	12 760.2	9485.6	0.67	1.11	0.66	0.50333	0.503	1.30	0.77
5	0.7299	0.9180	0.8240	0.80	1.13	11 403.1	19 031.4	14 283.1	0.60	1.25	0.80	0.50333	0.503	1.59	0.63
6	0.7499	0.9356	0.8428	0.80	1.12	14 677.3	23 252.2	18 202.7	0.63	1.24	0.89	0.50333	0.503	1.77	0.57
7	0.7265	0.8825	0.8045	0.82	1.11	17 111.5	22 945.9	19 981.7	0.75	1.17	0.25	0.50333	0.503	0.49	2.02
8	0.7236	0.8987	0.8111	0.81	1.12	19 836.7	23 302.2	20 358.8	0.85	1.03	0.46	0.50333	0.503	0.91	1.10
9	0.9292	0.8983	0.8354	1.03	0.90	32 296.4	28 132.5	23 968.7	1.15	0.74	0.81	0.50333	0.503	1.60	0.62
10+	0.7856	0.7889	0.7240	1.00	0.92	34 628.6	32 193.5	29 758.3	1.08	0.86	0.75	0.50333	0.503	1.49	0.67

*This table reflects the results from the simulations as recorded. For the purposes of determining degree of belief in membership, any simulated value > 1.00 (100% belief) was set to 1.00 (see Table 9).

Table 9. Consumption membership functions for *Lutjanus campechanus* (Poey) by age category

Age	Membership for max % consumption at the reef	Membership for foraging consumption
2	1.00/High + 0.96/Low	1.00/Great + 0.65/Small
3	0.79/High + 1.00/Low	0.74/Great + 0.97/Small
4	0.77/High + 1.00/Low	0.67/Great + 1.00/Small
5	0.80/High + 1.00/Low	0.60/Great + 1.00/Small
6	0.80/High + 1.00/Low	0.63/Great + 1.00/Small
7	0.82/High + 1.00/Low	0.75/Great + 1.00/Small
8	0.81/High + 1.00/Low	0.85/Great + 1.00/Small
9	1.00/High + 0.90/Low	1.00/Great + 0.74/Small
10+	1.00/High + 0.92/Low	1.00/Great + 0.86/Small

Table 10. Reef distance membership functions for each age class from Crystal Ball simulations

Age	Membership of reef location to major and minor distances
2	1.00/Major + 0.53/Minor
3	0.68/Major + 1.00/Minor
4	1.00/Major + 0.77/Minor
5	1.00/Major + 0.63/Minor
6	1.00/Major + 0.57/Minor
7	0.49/Major + 1.00/Minor
8	0.91/Major + 1.00/Minor
9	1.00/Major + 0.62/Minor
10+	1.00/Major + 0.67/Minor

In a similar manner, the membership functions were generated for all age groups (Table 9).

Next, the belief in the degree of membership was similarly accepted to be the ratio of the randomly generated distance for an age group in relation to the Maximum and Minimum reef distance based on the defined triangular distribution. Again, from Table 8 for Age 2 *L. campechanus*, the randomly generated reef distance of 0.95 exceeds the Maximum simulated reef distance value (i.e. has membership of 1) but the Minimum is only partially that of the reef distance designated for that age *L. campechanus*. Membership functions for Age 2 and all other ages of *L. campechanus* according to reef distances are given in Table 10.

Each linguistic variable was then defined from the membership functions in Table 10. Given previous research on reef spacing (Grove & Suno 1985; Strelcheck *et al.* 2005), and concentrating within a 1-km² area, the focus was on defining the Major (*M*) distance, not Minor (*N*). Therefore, Major distances (0.50 km

apart) between reefs as a function of Age of *L. campechanus* is:

$$M = 1.00/\text{Age}2 + 0.68/\text{Age}3 + 1.00/\text{Age}4 + 1.00/\text{Age}5 + 1.00/\text{Age}6 + 0.49/\text{Age}7 + 0.91/\text{Age}8 + 1.00/\text{Age}9 + 1.00/\text{Age}10+.$$

Membership of High and Low *P*-value and Great and Small consumption (g) according to the age of *L. campechanus* is as follows:

$$H = 1.00/\text{Age}2 + 0.79/\text{Age}3 + 0.77/\text{Age}4 + 0.80/\text{Age}5 + 0.80/\text{Age}6 + 0.82/\text{Age}7 + 0.81/\text{Age}8 + 1.00/\text{Age}9 + 1.00/\text{Age}10+.$$

$$L = 0.96/\text{Age}2 + 1.00/\text{Age}3 + 1.00/\text{Age}4 + 1.00/\text{Age}5 + 1.00/\text{Age}6 + 1.00/\text{Age}7 + 1.00/\text{Age}8 + 0.90/\text{Age}9 + 0.92/\text{Age}10+.$$

$$\begin{aligned}
G &= 1.00/\text{Age}2 + 0.74/\text{Age}3 + 0.67/\text{Age}4 \\
&\quad + 0.60/\text{Age}5 + 0.63/\text{Age}6 + 0.75/\text{Age}7 \\
&\quad + 0.85/\text{Age}8 + 1.00/\text{Age}9 + 1.00/\text{Age}10+. \\
S &= 0.65/\text{Age}2 + 0.97/\text{Age}3 + 1.00/\text{Age}4 \\
&\quad + 1.00/\text{Age}5 + 1.00/\text{Age}6 + 1.00/\text{Age}7 \\
&\quad + 1.00/\text{Age}8 + 0.74/\text{Age}9 + 0.86/\text{Age}10+.
\end{aligned}$$

Results

The triangular functions that were defined for each age for P -value, consumption (g) and reef distance (km) were set as assumptions with results generated as I and J functions based on a series of simulation runs as described previously. From Eqn (8)

$$I(A \subset B) = \inf_x \text{Max}\{1 - A(x), B(x)\}.$$

As such, the complements and the reef distance functions determine, for each age category, the maximum belief in the subset relationship of {High, Low} P -value, {Great, Small} consumption (g) value and combinations of both parameters to reef distance.

Continuing the example, based on the simulated values in Table 8, for Age 2 *L. campechanus* with total belief, 1.00 membership, in High percent maximum consumption at the reef, the complement ($1 - H$) would be 0, and the beliefs in the subset relationship to Major reef distance (H_M) would be {0, 1.00} with maximum belief = 1.00. Then, the value of $I(H_M)$ is the minimum belief over all values for ages 2 through 10+; 0.49 which occurs at the randomly generated reef distances of 0.25 km for age 7. Then, the minimum set for which the function $I(H_M)$ measures the degree to which the P -value (percent maximum consumption) is included (i.e. a factor) in Major reef distance (0.503 km) has belief of 0.49. All other I functions for Major (M) reef distance are calculated in this same manner such that:

$$\begin{aligned}
I(H \subset M) &= 0.49 & I(H \cap G \subset M) &= 0.74 \\
I(L \subset M) &= 0.49 & I(H \cap S \subset M) &= 0.79 \\
I(G \subset M) &= 0.49 & I(L \cap G \subset M) &= 0.74 \\
I(S \subset M) &= 0.49 & I(L \cap S \subset M) &= 0.97
\end{aligned}$$

From Eqn (9), $J(A\#B)$ measures the degree to which A intersects B

$$J(A\#B) = \text{Max}_x \text{Min}\{A(x), B(x)\}.$$

Again using the continuing example at Age 2, P -value belief is 1.00 in High, and belief is 0.96 in Major reef distance so that $\text{Min}\{1.00, 0.96\}$ is 0.96 for $J(H\#M)$ or

0.96 belief that High maximum consumption at the reef for this category intersects with Major reef distance. The actual $J(H\#M)$ is calculated as the maximum belief from the minimum observed at each category. This value for $J(H\#M)$ is 1.00 which is observed for Ages 2, 9 and 10+. This represents 100% belief that if percent maximum consumption is High at the reef then the reef distances are Major (0.503 km). For the data in Table 8, again restricting maximum belief to 1, the degree to which the fuzzy sets for P -value and consumption (food consumed) intersect M are:

$$\begin{aligned}
J(H\#M) &= 1.00 & J(H \cap G\#M) &= 1.00 \\
J(L\#M) &= 1.00 & J(H \cap S\#M) &= 0.86 \\
J(G\#M) &= 1.00 & J(L \cap G\#M) &= 0.96 \\
J(S\#M) &= 1.00 & J(L \cap S\#M) &= 1.00.
\end{aligned}$$

Therefore, for the above I and J functions determined from the 10 000 simulation runs in Table 8, the following certain and possible rules, respectively, can be written based upon a designated threshold of acceptance, α .

With a threshold of $\alpha = 0.95$, a certain rule for major reef distance is:

1. If realised consumption at the reef (P -value) is Low and food consumed (g) is Small, then reef distance should be Major (0.5033). (Certain with Belief = 0.97)

At $\alpha = 0.75$, a certain rule for major reef distance is:

2. If realised consumption at the reef (P -value) is High and food consumed (g) is Small then reef distance should be Major (0.5033). (Certain with Belief = 0.79)

Two other rules of lesser certainty are:

3. If realised consumption at the reef (P -value) is High and food consumed (g) is Great then reef distance should be Major (0.5033). (Certain with Belief = 0.74)

4. If realised consumption at the reef (P -value) is Low and food consumed (g) is Great then reef distance should be Major (0.5033). (Certain with Belief = 0.74)

With $\alpha = 0.95$, seven of the eight rules show strong belief in the possibility that reef distance should be 0.503 km.

The above results relate only to one series of simulated data given in Table 8. Based on the nine scenarios tested for reef distances from 0.01–0.50 km, to 0.50–0.95 km under the assumptions for Maximum and Minimum consumption at the reef (P -values) simulated as described previously using the bioenergetics modelling results for each age group and again based on the 10 000 simulation runs for each designated reef distance, the following was observed:

Certain rules: ($\alpha \geq 0.94$)

1. If realised consumption at the reef (P -value) is High, then reef distances should be 0.25–0.95 km. (Belief = 1.00)
2. If realised consumption at the reef (P -value) is Low, then reef distances should be 0.25–0.95 km. (Belief = 1.00)
3. If food consumed is Great, then reef distances should be 0.25–0.95 km. (Belief = 1.00)
4. If food consumed is Small, then reef distances should be 0.25–0.95 km. (Belief = 1.00)
5. If realised consumption at the reef (P -value) is High and food consumed (g) is Great, then reef distances should be 0.25–0.95 km. (Belief = 1.00)
6. If realised consumption at the reef (P -value) is High and food consumed (g) is Small, then reef distances should be 0.25–0.95 km. (Belief = 1.00)
7. If realised consumption at the reef (P -value) is Low and food consumed (g) is Great, then reef distances should be 0.25–0.95 km. (Belief = 1.00)

All of the above rules had strongest belief in reef distances >0.25 km apart. However, more refinement of distances but with lower belief was observed by the following:

8. If realised consumption at the reef (P -value) is Low and food consumed (g) is Small, then reef distances should be 0.50–0.95 km. (Belief = 0.985)
9. If realised consumption at the reef (P -value) is Low and food consumed (g) is Small, then reef distances should be 0.50–0.85 km. (Belief = 0.947)

Possible rules: ($\alpha \geq 0.95$)

All rules are possible for reef distances 0.01–0.50; 0.25–0.50; 0.35–0.50; and 0.45–0.50 and 0.50–0.95 km.

Discussion

The process of collecting data for evaluating bioenergetics models was considered difficult 20 years ago (Hewett & Johnson 1992). Supporting evidence, however, establishes that data sets for bioenergetics models can be developed if knowledge of observed growth rates, daily consumption levels, temperatures, and reasonable values of activity cost and caloric density for fish and their prey are available (Bajer *et al.* 2004b). The *L. campechanus* parameter values for age classes in this model were determined from data derived from tagging studies, diet analyses, site fidelity estimations and reproductive history (Patterson *et al.* 2001a, b; Strelcheck 2001; McCawley 2003; Woods 2003; Jackson *et al.* 2007; McCawley & Cowan 2007). Thus, the model provides an approximation of bioenergetics of and prey demand for *L. campechanus* based upon empirical data.

Combined results from multiple sources were used to provide age-specific predicted growth, consumption and respiration rates along with an estimate of total consumption for a single reef population, and thus the amount of prey required to sustain the population. The use of the Wisconsin Bioenergetics software to create a *L. campechanus* bioenergetics model provided a view into the dynamics within the snapper population on the reef systems (see Shipley 2008 for a complete description of the *L. campechanus* bioenergetics model).

The bioenergetics parameters when inputted into the fuzzy rule-based model showed that High consumption at the reef (P -value) with Great or Small foraging consumption (g) does not overly influence reef placement. Similarly, Low P -value and Great consumption (g) do not overly influence reef placement. The minimum distance that received perfect (100% belief) strength for any parameter or combination of parameters was reef placement of no closer than 0.25 km. However, with slightly lesser strength, Low consumption at the reef (P -value) and Small foraging consumption (g) placed reef distances at a minimum of 0.50 km with sufficient belief in the certainty of this relationship (belief of 0.985).

The high belief in the possibility of all parameters tested for each of nine distance scenarios supports that consumption at the reef {High, Low} and foraging consumption {Great, Small} appear to relate equally as possible influences upon reef distances from 0.01 to 0.95 km. However, the overriding factors in reef placement are Low consumption at the reef (P -value) and Small foraging consumption (g) which suggest that reef placement should be no closer than 0.50 km, preferably 0.50–0.95 km (belief = 98.5). The possibility results with perfect belief (1.00) for each range show that the two factors have a strong degree of relationship to reef locations. Therefore, the results of the fuzzy rough set modelling provide evidence that reef locations should be between 0.50 and 0.95 km, such that no more than two fit within a 1-km² area.

More than 20 years ago, Bohnsack (1989) suggested that it is likely that prey demand necessary to support observed fish biomasses on individual artificial reefs is high and cannot be equated with local prey production. Fish production may be dependent upon some critical density of artificial reefs (also see papers in *Fisheries*, April 1997), or at the very least, dependent upon fish movement off, or perhaps between, nearby reefs during foraging. Thus, determination of the effects of: (1) artificial reef size; (2) the spatial arrangement and density of artificial structures; and (3) proximity to natural habitats on the demographics of reef-associated fishes are neces-

sary steps in addressing the relative value of artificial reefs as fish habitat. As early as 1998, Bortone reiterated that ‘artificial reef location studies – as well as analyses of the impacts that large-scale environmental conditions have on the locally attracted fish assemblages – are long overdue’.

This research approaches the solution to artificial reef placement from the perspective of energetics. Fish gain energy through food; this energy is used for metabolic activities and growth (Adams & Breck 1990). If sufficient energy to promote reef fish population growth (in weight) in excess of losses of biomass due to mortality is available on an artificial reef, then the reef will produce new fish biomass (Ricker 1975). However, artificial reefs in the northern Gulf of Mexico are most probably energy sinks (Bortone 1998). Most of the trophic energy that comes to any reef is derived from the water column that passes by, or from the substrate on which the reef sits. Thus, the carrying capacity of a reef depends upon the amount of energy delivered as food items in the water column, and in the form of food captured from the surrounding benthic substrate (Bortone 1998). Depending upon reef size and the number of individuals present, McCawley and Cowan (2007) estimated that *L. campechanus* alone could consume 82–975 kg of prey per year on Alabama artificial reefs assuming a consumption/biomass ratio of 1.44 (FishBase; <http://fishbase.org/Summary/SpeciesSummary.php?ID=1423>), with only a small fraction (1.3%) of the prey species consumed being derived directly from the reef. Rather, most of the prey consumed was gathered from the surrounding soft bottom sediments. The bioenergetically derived consumption results for a *L. campechanus* population on a single reef in the Alabama shelf waters were less (16–26 kg yr⁻¹) than estimates by McCawley and Cowan (2007), undoubtedly owing to the latter’s failure to account for size structure. As such, it seems likely that fish foraging dynamics on artificial reefs may be governed by resource mosaic dynamics as described by Lindberg *et al.* (1990) for stone crabs.

The determination of a bioenergetics based on environmental factors and a synthesis of field research with a fuzzy set-based model may prove useful for future management practices and builds on the concept that predicting how management actions can influence an ecosystem requires simulation modelling; a classic use of ecosystems models (Minns 1992) whether from the perspective of a crisp or fuzzy research focus. A benefit of the fuzzy rule-based model presented herein is that while the technical data were derived by bioenergetics modelling or obtained through field research, once available, simulation and subsequent

analysis of the membership functions do not require extensive technical knowledge of the decision maker. Indeed, as a spreadsheet applicable modelling process, management decisions can be made in the field. Using a readily available simulation package that works with Excel, the simulations were easily conducted on a laptop computer. The model can be readily updated as necessary to react to Sakuramoto’s (1995) suggestion that refinement of the rules is always a consideration, and accumulation of sufficient data upon which to base the simulation of distributions of ecological variables should be ongoing. The model is, therefore, applicable to other fisheries management decisions for species other than *L. campechanus*. The results, likewise, have relevance to the discussion of artificial reef placement in bodies of water outside of the Alabama shelf region of the Gulf of Mexico.

Acknowledgments

The authors acknowledge the review of the fuzzy rough set rule-based model by Dr Margaret F. Shipley, University of Houston Downtown. The authors also acknowledge the assistance of the manuscript’s referees towards improving the quality, accuracy and readability of the research presented.

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